VIMADES

Viabilité, Marchés, Automatique et Décision

Robustesse, résistance et résilience dans les systèmes naturels

Tychastic Viability

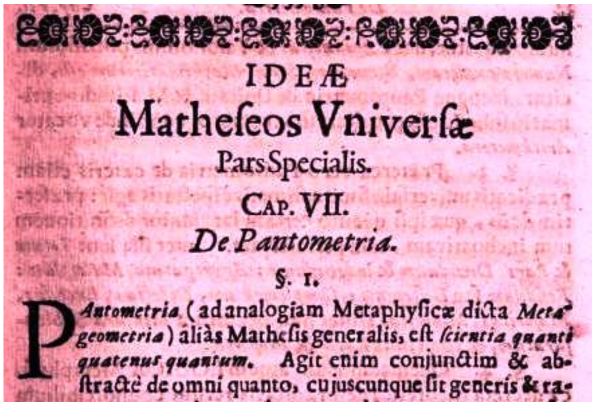
Jean-Pierre Aubin

SFBT
Sociéte Francophone de Biologie Théorique
June 11-13, 2012

1 “Robustness, Resistance and Resilience” Involve

1. (calendar) time and (variable) duration;
2. constraints and targets (and their evolution governed by morphological equations)
3. uncertainty games
   (a) tychastic uncertainty (stochastic is a too particular case)
   (b) contingent uncertainty: continuous or impulsive (discontinuous);
   (c) how contingent uncertainty offsets tychastic uncertainty);
4. either forecasting or anticipation ;
5. cooperation leading to connexionnist and Lamarckian complexity.

1. **In life sciences, evolution is *never at equilibrium* (stationary state). No need that it converges to it (asymptotic stability),**
2. **Chaos, a property of deterministic system, is not fit to represent a nondeterministic behavior of living systems which struggle to remain as stable (and thus, "non chaotic") as possible;**
3. **that the future cannot be predicted, but anticipated by retroaction processes, and that initial conditions should be replaced by terminal conditions for learning from the past;**
4. ***si la vie avait un but, ce ne serait pas la vie*;**
5. **that those human brains should complement it by another and more recent principle, *adaptation of transient evolutions to environments*.**



IDEÆ

Matheseos Vniversæ

Pars Specialis.

Cap. VII.

De Pantometria.

§. 1.

Pantometria (ad analogiam Metaphysicæ dicta *Meta-geometria*) aliàs Mathesis generalis, est *scientia quanti quatenus quantum.* Agit enim conjunctim & abstractè de omni quanto, cujuscunque sit generis & ra-

The **Pantometric** scandal: **Quantitative versus qualitative evaluation. Cognitive sciences doesn't accept that rationality** of human brains can be reduced to **maximization of utility functions**: Poincaré : "***Satisfaction is thus a magnitude, but not a measurable magnitude***"

6. **Intertemporal optimality**, a creation of the human brain to explain some physical phenomena requires a **Decision Maker, Optimality Criterion, Knowledge of the future, of the Objective** and **Determinism** once the initial condition is fixed.

7. **Uncertainty in natural systems** cannot be mathematically captured only by probabilities and stochastic processes, in the legacy of Ingenhousz (1785),

1. **Direct Approach. It consists in studying properties of evolutions governed by an evolutionary system: gather the largest number of properties of evolutions starting from each initial state. It may be an information both costly and useless, since our human brains cannot handle simultaneously too many observations and concepts.**

2. **Inverse Approach. A set of prescribed properties of evolutions being given, study the (possibly empty) subsets of initial states from which**

   (a) **all evolutions starting from it satisfy these prescribed properties (tychastic uncertainty).**

   (b) **starts at least one evolution governed by the evolutionary system satisfying the prescribed properties (contingent uncertainty),**

Simplifying complexity

***Ockham*'s razor prescribes is a "law of parsimony" stating that an explanation of any phenomenon should make as few assumptions as possible, and to choose among competing theories the one that postulates the fewest concepts.**

Paradox: "simple" physical phenomena are explained by more and more sophisticated and abstract mathematics, whereas "complex" phenomena of living systems use, most of the time, relatively rudimentary mathematical tools. This is the result of an "abstraction process", which is the (poor) capability of human brains that selects among the perceptions of the world the few ones from which they may derive logically or mathematically many other ones.

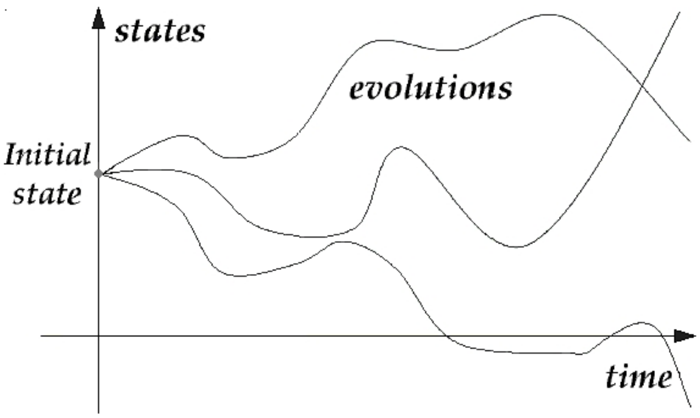
***Simplifying complexity* should be the purpose of an emerging science of complexity, if such a science will emerge beyond its present fashionable status.**

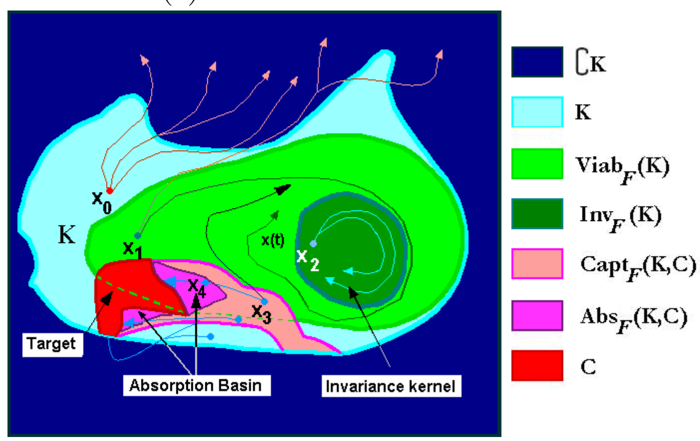
2 The Viability Problem:

Chance   *and*   *Necessity*

$$\Updownarrow \qquad\qquad\qquad \Updownarrow$$

$$x'(t) \in F(x(t)) \quad \& \quad x(t) \in K$$





Regulation maps governing the evolution of evolutions viable in one environment always or until they capture a target in finite time.

Resilience

1. **Given a current subset of states C (an equilibrium $C := \{\overline{e}\}$) Holling (1973) defines resilience as the capacity of a system to undergo "disturbance" or tychastic perturbations and returns to C;**

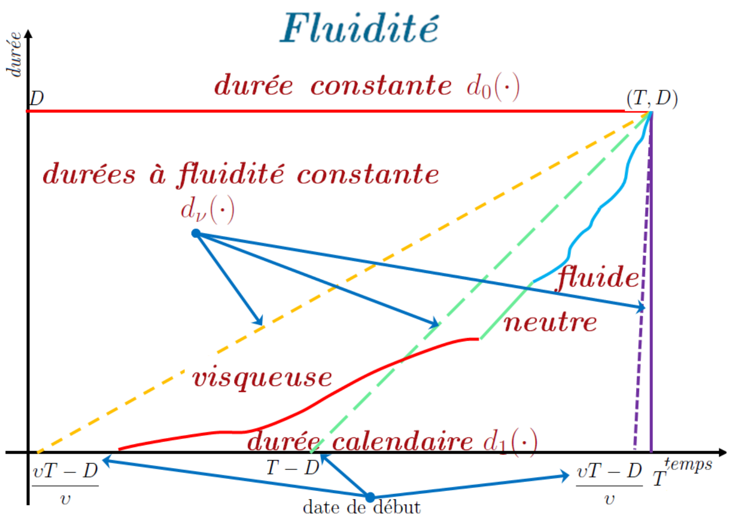
2. **The concept of resilience can be described by the concept of Tychastic Absorption Basin: for all evolutions of tyches, the evolution of the state returns to C in finite time.**

3. **Sophie Martin (2005) Resilience mesures the inverse of the cost associated with the effort necessary to maintain or restore the property of interest after a disturbance, related to invariant absorption basins.**

   (a) state system not necessary in the vicinity of an equilibrium

   (b) emphasis put on the set where this property holds.

3 The times of Time: Duration, Repetition, etc.



1. **The Spatial Metaphor of time by its position in $t \in \mathbb{R}$.**
2. ***Variable duration:* "*duration space*" $\mathbb{R}_+$;**
3. ***Variable duration:* $d(\cdot) := t \in$ R $\mapsto d(t) := d_{(T,D)}(t) \in \mathbb{R}_+$ satisfying $d'(t) \geq 0$ and $d(T) = D$.**

1. ***Aperture of the temporal window* of $d(\cdot)$:**

$$\Omega(d(\cdot)) := \Omega_{T,D}(d_{T,D}(\cdot)) := \inf_{\{\omega\geq 0 \text{ such that } d_{T,D}(T-\omega)=0\}} \omega \in [0,+\infty] \tag{1}$$

2. ***Temporal window* $[T-\Omega(d_{T,D}(\cdot)),T]$.**

Examples

1. ***chronological time*: is the evolution $t \in [\mathbf{O},T] \mapsto d_{D+\mathbf{O},D}(t) := \max(t-\mathbf{O},0)$ $d(\mathbf{O}) = 0$ to $d(T) = T-\mathbf{O} = D$ on the time window $[\mathbf{0}, D+\mathbf{O}]$ of aperture equal to D, where *origin of time* $\mathbf{O}$, durations $D \geq 0$ and future times $T := D+\mathbf{O}\mathbb{R}$.**
   Chronological time plays the role of a *"numéraire of evolutions"*

2. ***calendar age* $t \in [T-D,T] \mapsto d_1(t) := \max(0, t-(T-D))$ from $d_1(T-D) = 0$ to $d_1(T) := D$. Durations space $D \in \mathbb{R}_+$ are ages, *which has an origin* (age equal to 0 at birth)**

Variable durations $d(\cdot) : t \mapsto d(t) \geq 0$ are no longer prescribed, but *chosen* among available ones and *regulated*, providing as a by product the *temporal windows* on which they evolve:

$$(d'(t), x'(t)) \in F(t, d(t), x(t)) \subset \mathbb{R}_+ \times X$$

$d(T) \ = \ D$ and $x(T) = x$ **(terminal conditions)**

Looking backward on the past temporal window $[T - \Omega(d_{T,D}(\cdot)), T]$ associated with the governed evolution $d(\cdot)$ chaperoning the evolution $x(\cdot)$.

Knowledge of the Future

In physics, *the horizon T and the origin of time $\mathbf{O}$ are fixed* and the state is governed by "predictive systems" $x'(t) \in F(x(t))$ satisfying $x(\mathbf{O}) = x$ at the beginning of the future *prescribed chronological window* $[0, T]$.

The knowledge of the future assumes some regularity (periodicity, cyclicity, etc.) requiring predictions, or *demands experimentation*: "translate" a temporal window back and forth for deducing the the evolution is the translated.

Living systems are *myopic*. Instead of taking into account the future, their evolutions are certainly constrained by their *history*. The systems are irreversible, *their dynamics may disappear and cannot be recreated*, forbidding any insight into the future.

Anticipation or Prediction?

Predict or *anticipate* the future, i.e., extrapolate past evolutions and to constrain in the last analysis the evolution of the system as a ***function of its history***. However, to quote ***Paul Valéry***,

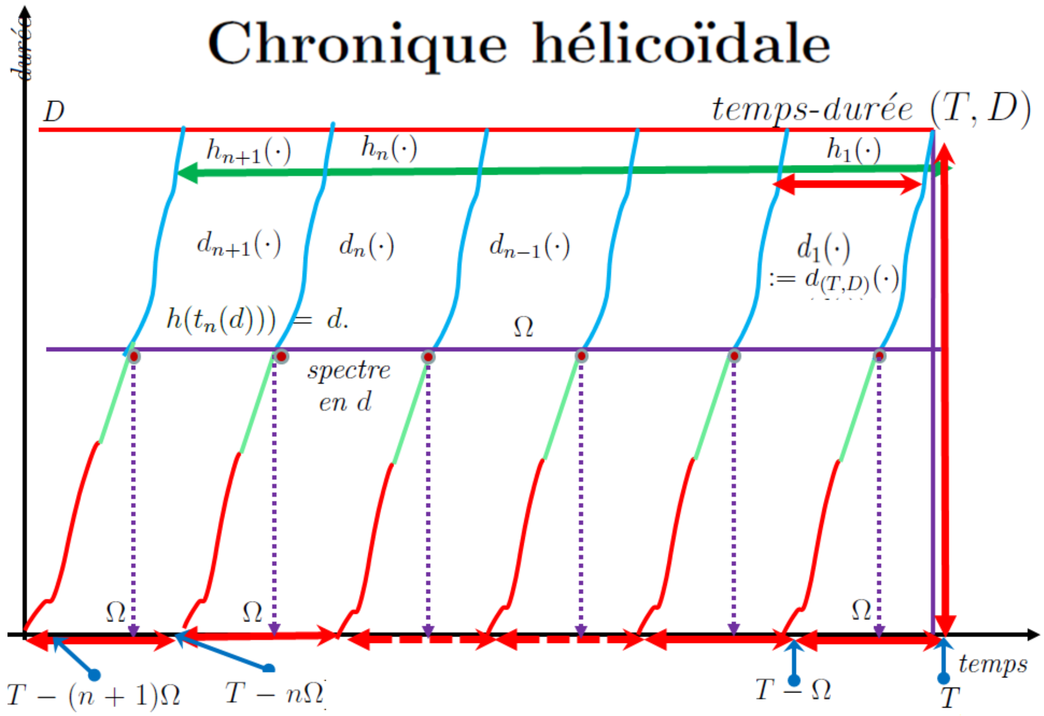
"***Forecasting is a dream from which reality wakes us up***".

The choice (even conditional) of the optimal controls is made *once and for all* at some initial time, and thus *cannot be changed at each instant so as to take into account possible modifications of the environment of the system*, thus forbidding ADAPTATION to viability constraints.

For systems involving living beings, there is not necessarily an actor governing the evolution of regulons according to the above prerequisites.

The choice of criteria is open to question even in static models, even when multicriteria or several decision makers are involved in the model.

Repetition, leads to the concept of "helicoidal time", mixing chronological time (rod) and cyclic variable durations (snake).



4 Dealing with Uncertainty: Tyches and/or Regulons

The mathematical description of the evolutions of states depend upon parameters which differ according to the problems and to questions asked:

1. **"tyches" or disturbances, perturbations under which nobody has any control or influence.**

2. **"controls", whenever a controller or a decision maker "pilots" the system by choosing the controls, as in engineering,**

3. **"regulons" or regulatory parameters in those living systems where no identified or consensual agent acts on parameters of the system,**

   (a) **Regulons can be plain vectors;**

   (b) **Regulons can be matrices and tensors: connexionnist complexity for regulating networks.**

Tychastic Uncertainty

Charles Peirce introduce tychastic evolution in 1893 in Evolutionary Love for describing the evolution of a system dependent on tyches arising without any statistical regularity.

1. Tyches are identified (velocities or rates of the uncertain variables) which can then be used when the realizations of events are actually observed and known at each date during the evolution. They range over a state-dependent ***tychastic map*** $(t, x) \rightsquigarrow V(t, x)$ which describes it.

2. For this reason, the results are computed in the worst case (***eradication of risk*** instead of its ***statistical evaluation***);

3. required properties are valid for "all" evolutions of tyches $t \mapsto v(t) \in V(t, x(t))$ instead of ***constant*** ω's.

***Tyche*.** **Uncertainty without statistical regularity can be translated mathematically by parameters on which actors, agents, decision makers, etc. have no controls. These parameters are often perturbations, disturbances (as in "robust control" or "differential games against nature") or more generally, *tyches* (meaning "chance" in classical Greek, from the Goddess Tyche whose goal was to disrupt the course of events either for good or for bad.**

Tyche became "Fortuna" in Latin, "rizikon" in Byzantine Greek, "rizq" رزق in Arabic (with a positive connotation in these three cases). "reaction, change", 应 变 , translates the concept of tychasticity.

The larger the tychastic map, the smaller the invariance kernel.

How to Offset Tychastic Uncertainty?
La nécessité du Hasard (Alain Pavé) :
Contingent Uncertainty and Redundancy

1. **By introducing a reservoir of controls or regulons (contingent map $x \rightsquigarrow U(x)$);**

2. **building a retroaction map $(t, x) \mapsto \widetilde{u}(t, x)$ independent of the tyches.**

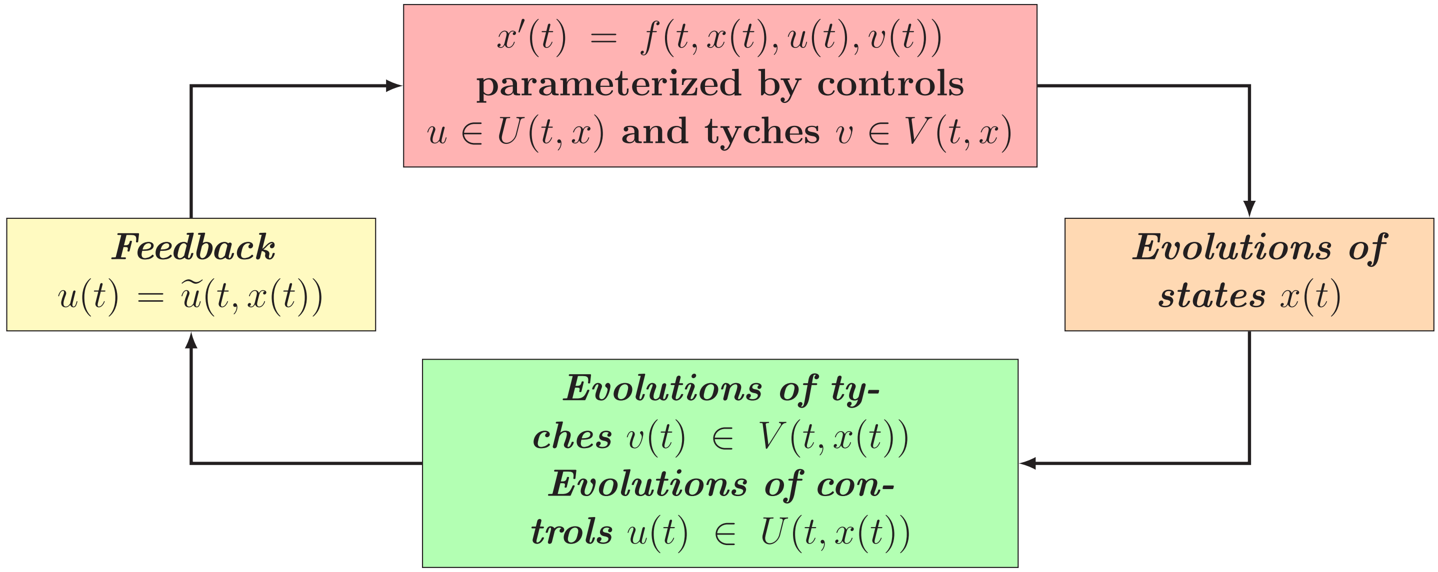
Guaranteed viability kernel: the union of the invariance kernels associated with each retroactions $\widetilde{u}$.

The size of the contingent map describes the *redundancy*: *The larger the contingent map, the larger the guaranteed viability kernel.*

The word contingent comes from the Latin verb contingere, to arrive unexpectedly or accidentally. Leibniz: "Contingency is a non-necessity, a characteristic attribute of freedom."

未 定 , "no, necessary", translates contingent.

Regulated Tychastic System: Evolutionary Engine



5 Punctuated Equilibria and the Inertia Principle

Darwin **added the sixth edition of is celebrated book the sentence**

“*and lastly, although each species must have passed through numerous transitional stages, it is probable that the periods, during which each underwent modification, though many and long as measured by years, have been short in comparison with the periods during which each remained in an unchanged condition*”

(personal communication by *Jim Murray*).

In the absence of an actor piloting the regulons, or by assuming that this actor is

***myopic, explorer and lazy, opportunistic and conservative*,**

We cannot assume any longer that the regulons are chosen to minimize an intertemporal criterion.

We may assume instead that regulons evolve as "slowly" as possible because the change of regulons (or controls in engineering) is costly, even very costly.

Evolutions under constant coefficients, which do not evolve at all, may not satisfy required properties. Then the question arises to study

when, where and how

coefficients must cease to be constant and start to "evolve" in order to satisfy the required property, for instance.

In this case, their status of "coefficients" is modified, and they become controls or regulons, according to the context (engineering or life sciences where the problem is set).

Inertia Principle

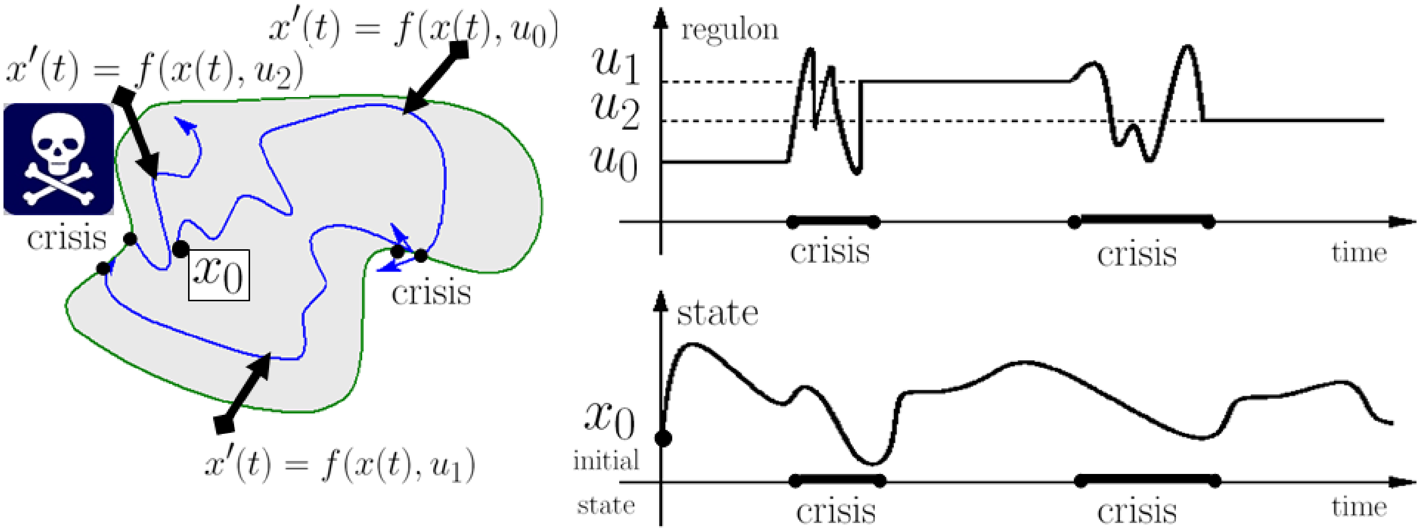
Whenever the viability property is concerned, we shall give a name to this phenomenon which seems to be shared by so many systems dealing with living beings: In a loose way,

the *inertia principle* states that the *"regulons" of the system are kept constant as long as possible and changed only when viability or inertia is at stake*.

The inertia principle provides a mathematical explanation of the emergence of the concept of *punctuated equilibrium* introduced in paleontology by *Nils Eldredge* and *Stephen J. Gould* in 1972.

"Heavy evolutions" are evolutions minimizing AT EACH INSTANT (and not in an intertemporal way) the velocity of their regulons.

Heavy evolutions provide the simplest examples of evolutions satisfying the inertia principle.


$x'(t) = f(x(t), u_0)$
$x'(t) = f(x(t), u_2)$
crisis
x_0
crisis
$x'(t) = f(x(t), u_1)$
regulon
u_1
u_2
u_0
crisis
crisis
time
state
x_0
initial
state
crisis
crisis
time

6 Complexité Connexioniste

Paul Henri Thiry, baron d'Holbach, Système de la nature **(1750):**

1 ***"Enfin, si tout est lié dans la nature, si tous les mouvements y naissent les uns des autres quoique leurs communications secrètes échappent souvent à notre vue, nous devons être assurés qu'il n'est point de cause si petite ou si éloignée qui ne produise quelque fois les effets les plus grands et les plus immédiats sur nous-mêmes.***

C'est peut-être dans les plaines arides de la Lybie que s'amassent les premiers éléments d'un orage qui, portés par les vents, viendra vers nous, appesantira notre atmosphère, influera sur le tempérament et les passions d'un homme que ses circonstances mettent à la portée d'influer sur beaucoup d'autres, et qui décidera, d'après ses volontés, du sort de plusieurs nations".

Indice de complexité (connexionniste)

Le problème est de déceler des *liens insoupçonnés* entre des composantes d'un système qui s'établissent ou se renforcent afin de maintenir la viabilité du système étudié.

Ces *liens* $w_i^j(t)$ entre les variables sont décrits par une matrice de connexion $W(t) := (w_i^j(t))$, elle-même appelée à évoluer.

Un système est déconnecté (ou autonome, libre, décentralisé, etc.) si la "matrice de connexion" est la matrice identité I dont tous les liens liant chaque variable aux variables distinctes sont nuls.

À chaque instant $t \geq 0$, la distance

$$\|W(t) - \mathbf{I}\|$$

entre la matrice de connexion $W(t)$ et la matrice identité I est d'indice de complexité (connexionniste). Plus grande est cette distance, plus grand est cet indice de complexité connexionniste, plus complexe est le système.

Formulation mathématique

Système décentralisé

$$\forall\, i = 1, \ldots, n, \quad x_i'(t) \; = \; g_i(x_i(t)) \text{ ou } x'(t) \; = \; g(x(t))$$

soumis à des contraintes de viabilité

$$\forall\, t \geq 0, \quad h(x_1(t), \ldots, x_n(t)) \; \in \; M \text{ ou } h(x(t)) \; \in \; M$$

Corrections

1. **en "soustrayant" des "multiplicateurs de viabilité" ("prix" virtuels)**

$$x'(t) \; = \; g(x(t)) - p(t) \textbf{ décentralisation par les prix}$$

2. **en connectant les dynamiques des agents par des matrices**

$$x_i'(t) \; = \; \sum_{j=1}^{n} w_i^j(t) g_j(x_j(t)) \text{ ou } x'(t) \; = \; W(t) g(x(t))$$

Toute évolution viable gouvernée par le système corrigé par les prix est également gouvernée par le système corrigé par les matrices de connexion.

La réciproque est fausse, mais les évolutions viables régies par le système

$$x'(t) = W(t)g(x(t))$$

minimisant *à chaque instant* (de façon myope) l'indice de complexité connexionniste $\|W(t) - \mathbf{I}\|$ *sont les mêmes* que celles régies par le mécanisme

$$x'(t) = g(x(t)) - p(t) \text{ de décentralisation par les prix}$$

minimisant *à chaque instant* la norme $\|p(t)\|$.

7 Historic Differential Inclusions : a Forecasting Tool

Histories are evolutions $\varphi \in \mathcal{C}(-\infty, 0, \mathbf{R}^n)$ defined for negative times, which plays the role of a state space.They are the inputs of differential inclusions with memory

$$x'(t) \in F(\kappa(-t)x(\cdot)) \tag{2}$$

where

$$\forall \, \tau \leq 0, \;\; (\kappa(-t)x(\cdot))(\tau) \; := \; x(t+\tau)$$

and $F : \mathcal{C}(-\infty, 0; \mathbf{R}^n) \rightsquigarrow \mathbb{R}^n$ is a set-valued map defining the dynamics of history dependent differential inclusion.

The addition operator $\varphi \mapsto \varphi + h\psi$ is replaced by the *concatenation operator* $\diamondsuit_h$ associating with each history $\varphi \in \mathcal{C}(-\infty, 0; \mathbf{R}^n)$ the function $\varphi \diamondsuit_h \psi \in \mathcal{C}(-\infty, 0; \mathbf{R}^n)$ defined by

$$(\varphi \diamondsuit_h \psi)(\tau) \; := \; \begin{cases} \varphi(\tau + h) & \text{if} \;\; \tau \in]-\infty, -h] \\ \varphi(0) + \psi(\tau + h) & \text{if} \;\; \tau \in [-h, 0] \end{cases}$$

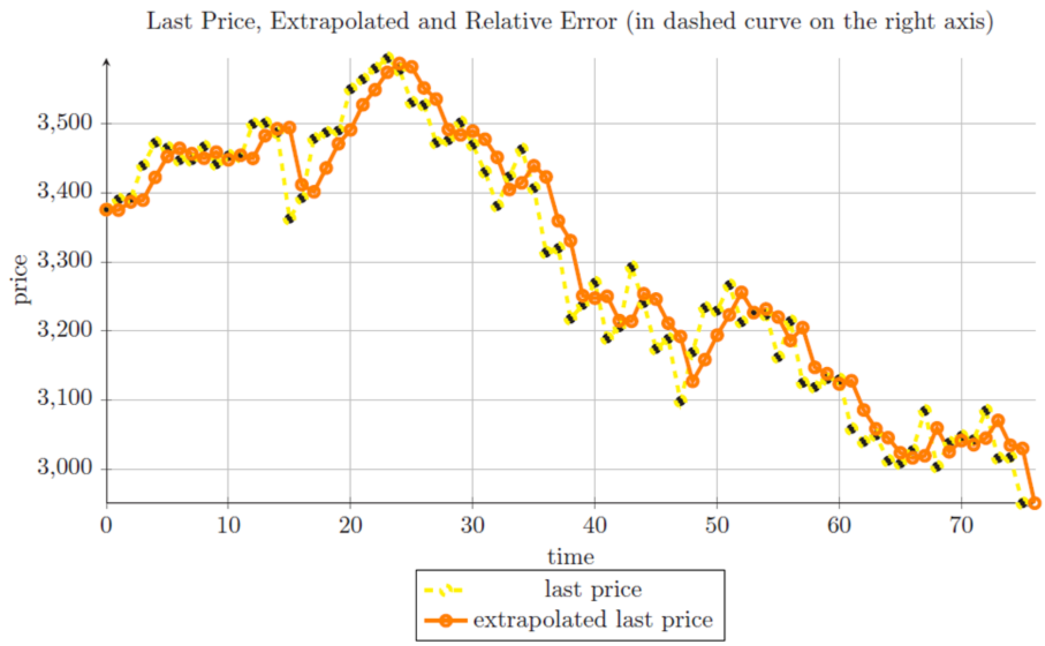
We can capture the concept of trends by the derivatives of histories and introduce trend dependent differential inclusions

$$x'(t) \in F\left((D^p(\kappa(-t)x(\cdot)))_{|p|\leq m}\right) \quad (3)$$

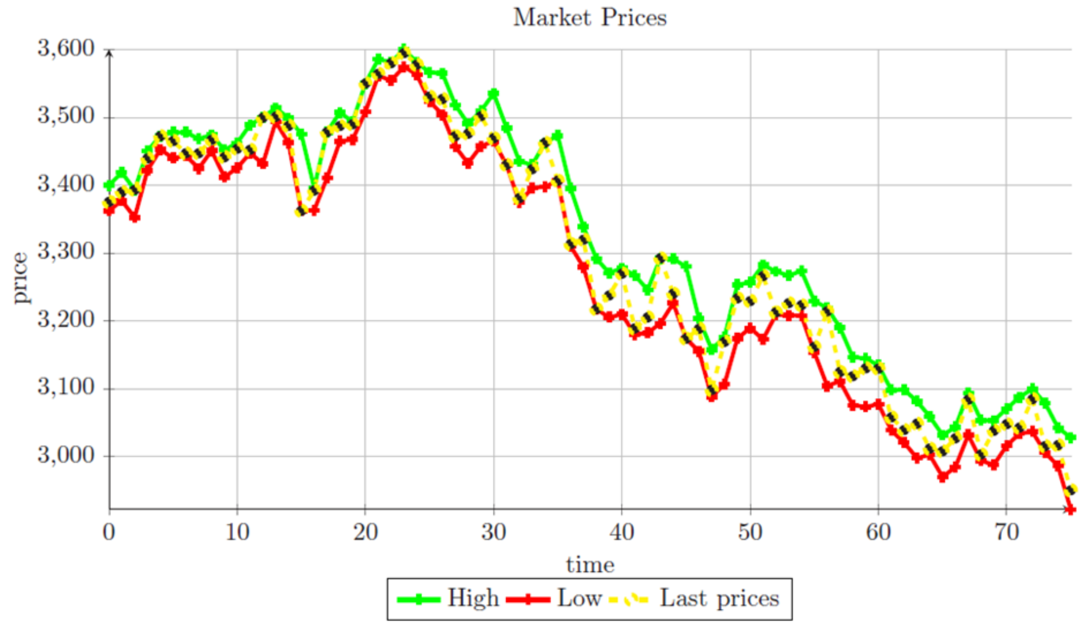
in order to take into account not only the history of an evolution, but its "trends".

The *VIMADES Extrapolator* (based on Laurent Schwartz distributions) is an example of history dependent differential inclusion which bypasses the use of a "volatilimeter" by extrapolating each history dependent al (past) evolutions of upper bounds (*HIGH*) and lower bounds (*LOW*) of the underlying prices

CAC40-from2012-02-14to2012-06-01-VAVCR



CAC40-from2012-02-14to2012-06-01-VAVCR



Historic or Clio Viability

The history dependent environments are subsets $\mathcal{K} \subset \mathcal{C}(-\infty, 0; \mathbf{R}^n)$ of histories.

Actually, the first "general" viability theorem was proved by Georges Haddad in the framework of history dependent differential inclusions at the end of the 1970's by Georges Haddad.

It requires a specific calculus of "Clio tangent sets" to an history dependent environment and of Clio derivatives of history dependent maps.

Let a history dependent functional $\mathbf{v} : \varphi \in \mathcal{C}(-\infty, 0, \mathbf{R}^n) \mapsto \mathbf{v}(\varphi) \in \mathbb{R}$.

We define the concept of *Clio derivative* by taking the limits of "differential quotients"

$$\nabla_h \mathbf{v}(\varphi)(\psi) \ := \ \frac{\mathbf{v}((\varphi \diamondsuit_h \psi)) - \mathbf{v}(\varphi)}{h} \in \ X \ := \ \mathbb{R}^n$$

for obtaining

$$D\mathbf{v}(\varphi)(\psi) \ := \ \lim_{h \to 0+} \nabla_h \mathbf{v}(\varphi)(\psi) \ \in \ X \ := \ \mathbb{R}^n$$

if it exists and is linear and continuous on $\mathcal{C}(-\infty, 0, \mathbf{R}^n)$ with respect to ψ. Then the gradient of v at φ is an element of the dual $\varphi \in \mathcal{C}(-\infty, 0, \mathbf{R}^n)^\star$ of $\mathcal{C}(-\infty, 0, \mathbf{R}^n)$, i.e., a (Radon) vector measure on $]-\infty, 0]$.

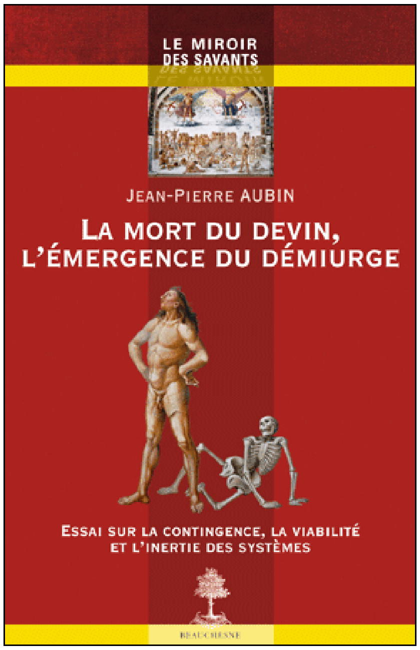
In the general case, stochastic uncertainty is described by a sample space Ω (of all possible outcomes), filtrations $\mathcal{F}_t$ of events at time t, the probability $\mathbb{P}$ assigning to each event its probability (a number between 0 and 1), a Brownian process $B(t)$, a drift $\gamma(S)$ and a volatility $\sigma(S)$: $dS(t) = \rho(S(t))dt + \sigma(S(t))dB(t)$.

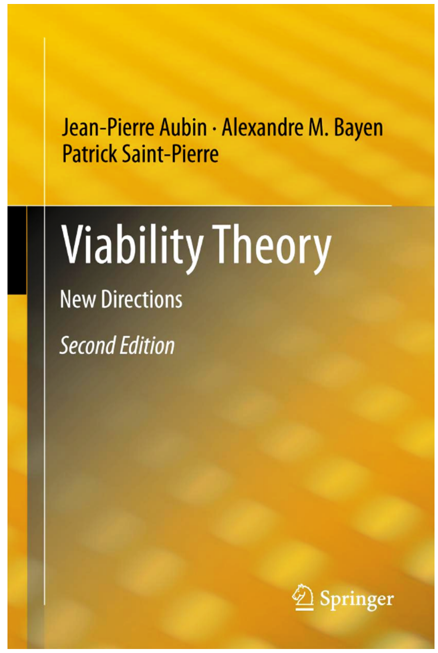
1. **The sample sets and the random events are *not explicitly identified* (in practice, one can always choose the space of all evolutions or the interval $[0, 1]$ in the proofs of the theorems). Only the drift and volatility are assumed to be explicitly known.**

2. **Stochastic uncertainty *does not study the "package of evolutions"* (depending on $\omega \in \Omega$), but *functionals over this package*, such as the different moments and their statistical consequences (averages, variance, etc.) used as evaluation of risk. They deal with the dual of the space of evolutions and on spaces of functionals on these evolutions. Even though in some cases, Monte-Carlo methods provide an approximation of the set of evolutions (for constant ω), there is no mechanism used for selecting the one(s) satisfying such or such prescribed property;**

3. **Required properties are valid for *"almost all" constant* ω.**

4. **Stochastic differential equations provide only the expectation of the package of evolutions, but do not allow to *select the right one* whenever, for every time $t > 0$, the effective realization ω (which then, depends on time), is known. This excludes a direct way to regulate the system by assigning to each state the proper ω, which, in this case, would depend on t, and thus, not part of an approximated set of evolutions computed by Monte-Carlo type of methods.**


LE MIROIR
DES SAVANTS
JEAN-PIERRE AUBIN
LA MORT DU DEVIN,
L'ÉMERGENCE DU DÉMIURGE
ESSAI SUR LA CONTINGENCE, LA VIABILITÉ
ET L'INERTIE DES SYSTÈMES


Jean-Pierre Aubin · Alexandre M. Bayen
Patrick Saint-Pierre
Viability Theory
New Directions
Second Edition
Springer